Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 1, pp. 59–62, 2002

EVALUATION OF THE ENERGY EFFICIENCY OF HEAT ADDITION UPSTREAM OF THE BODY IN A SUPERSONIC FLOW

A. F. Latypov and V. M. Fomin

UDC 533.6.011.5

Criteria for evaluating the energy efficiency of heat addition upstream of the body in a supersonic gas flow are obtained. Based on the functional objectives of flying vehicles and the thermodynamic model of the process, estimates are obtained for missile- and aircraft-type vehicles. The minimum Mach numbers at which heat addition upstream of the body is reasonable are evaluated. The increase in the flight range in the cruising regime for an aircraft-type vehicle and on the active trajectory for a missile-type vehicle is evaluated. Estimates for fuel economy in launching an aerospace plane into an Earth orbit are given. It is shown that a significant part of the fuel should be spent on producing energy for gas heating in order to obtain a noticeable effect. The minimum necessary "efficiency" of conversion of the fuel energy into the gas-heating energy is evaluated.

Numerous experimental studies of aerodynamic characteristics of hypersonic flying vehicles (HFV) show that the maximum lift-to-drag ratio in the hypersonic range of velocities is $K_{\max} \approx 4$ (see Fig. 1 in [1]). This value cannot be increased by aerodynamic design of HFV configurations. Therefore, much attention is currently paid to solving the problem of active flow control by means of an energy and/or force action on the incoming flow, in particular, by heat addition upstream of the body in a supersonic flow. This problem was considered in many papers (see, for instance, [2–5]). In technical implementation, laser and microwave radiation and an electric discharge are supposed to be used. The effect of drag reduction is mainly associated with a decrease in the free-stream gas density, which is confirmed by calculations [6–8] and direct measurements [9–1]. Additional effects are possible due to the changes in flow regime because of the decrease in the Mach number, variation of the Reynolds number, and flow ionization. It follows from the results of [5, 12–16], where gas dynamics of a wake flow behind an oscillating source of heat is studied, that the static pressure in the wake becomes rather rapidly equal to the ambient pressure. Most theoretical and experimental works deal with the problem of drag reduction. Gogish and Dashevskaya [17] demonstrated theoretically a significant effect of the stepwise distribution of the free-stream temperature on the lift force by an example of a hypersonic gas flow around an airfoil. It was found that the gliding regime is optimal under the condition of the maximum lift-to-drag ratio.

The efficiency of heat addition in a steady flight is traditionally evaluated as [18–20]

$$\eta = (A_0 - A)/Q, \tag{1}$$

where A_0 is the initial thrust power, A is the thrust power under the heat action, and Q is the power of heat addition. The efficiency η does not take into account the overall energy balance and the functional purpose of the flying vehicle (FV). In the present work, the efficiency is evaluated with regard for these factors.

Mathematical Model. It is assumed that heat addition in the incoming flow is performed at constant values of pressure and velocity, so that an infinite heat wake is formed with the following parameters upstream of the body:

$$P = P_\infty, \qquad V = V_\infty, \qquad T_\infty/T = \rho/\rho_\infty = F_\infty/F_0 = \varepsilon.$$

Here P is the pressure, V is the velocity, T is the temperature, F is the wake cross section, ρ is the density, and F_0 is the FV mid-section, and ε is a prescribed parameter; the subscript ∞ refers to gas parameters at infinity.

Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 1, pp. 71–75, January–February, 2002. Original article submitted September 24. 2001.

The heat power of the wake is

$$Q = \rho_\infty V_\infty F_0 c_p T_\infty (1 - \varepsilon),$$

where c_p is the heat capacity of air at constant pressure. It is also assumed that the flight of a vehicle possessing a high lift force occurs in a gliding regime: the lift force Y is mainly generated by the lower FV surface exposed to an undisturbed air flow:

$$Y_0 = c_y^0 q_\infty^0 S.$$

Here c_y^0 is the initial lift coefficient, $q_\infty^0 = \rho_\infty V_\infty^2/2$ is the dynamic pressure, and S is the planform area of the flying vehicle.

The drag may be approximately represented as a sum of two components: 1) longitudinal component of the normal force acting on the lower surface $X_1 = c_{x1}^0 q_\infty^0 S$; 2) drag of the body located in the heat wake $X_2 = c_{x2} q_\infty S$ ($q_\infty = \varepsilon q_\infty^0$). In a supersonic range of velocities and low angles of attack α, the relation $c_{x1}^0 = c_y^0 \alpha$ is valid; for the second term, we assume that $c_{x2} = c_{x2}^0 \xi$, where ξ is a coefficient that takes into account the change in the drag coefficient due to the possible change in the flow regime indicated above. Then, we have

$$X = (c_{x1}^0 q_\infty^0 + \xi c_{x2}^0 q_\infty) S.$$

Under the above assumption on drag components for the initial regime, we obtain

$$\bar{c}_{x2}^0 = c_{x2}^0 / c_x^0 = 1 - \alpha K_0,$$

where $c_x^0 = c_{x1}^0 + c_{x2}^0$ and K_0 is the lift-to-drag ratio in the initial regime. Under a thermal action, we obtain the following estimate for the relative lift-to-drag ratio:

$$1/\bar{K} = K_0/K = 1 - (1 - \varepsilon\xi)\bar{c}_{x2}^0.$$

The efficiency of heat addition estimated by Eq. (1) may be represented in the form

$$\eta = (k-1) c_x^0 \mathrm{M}_\infty^2 / 2$$

(k is the ratio of specific heats). This expression ignores some rather essential parameters of the process: degree of gas heating, FV lifting properties, engine performance, and efficiency of conversion of the fuel energy into the radiation energy. The possibility of using the efficiency η for an unlimited increase in the flight Mach number is not clear either.

Cruising Flight Regime of an Aircraft. In the cruising flight regime, the equations of motion have the form

$$\dot{L} = V_\infty, \quad \dot{m} = -R/I - Q/(\eta_Q \mathrm{Hu}), \quad R = X_0/\bar{K}, \quad mg = Y_0,$$

where L is the flight length, R is the engine thrust, m is the aircraft mass, I is the specific impulse of the engine, Hu is the calorific value of the fuel, and η_Q is the coefficient of conversion of the fuel energy into the energy absorbed by air. We obtain the following expression for the flight length:

$$L = -\eta_L \mathrm{Br} \ln(1 - g_T), \qquad \mathrm{Br} = V_\infty K_0 I / g,$$

$$\frac{1}{\eta_L} = \frac{R V_\infty + Q/\eta_Q}{R_0 V_\infty} = \frac{1}{\bar{K}} + \frac{1-\varepsilon}{\eta_Q} \frac{\bar{F}_0}{c_x^0 (k-1) \mathrm{M}_\infty^0 / 2} \frac{I a_\infty}{\mathrm{Hu}}.$$

Here Br is the Brege coefficient for the initial flight regime, a_∞ is the free-stream velocity of sound, and η_L is a coefficient, which shows the increase in the flight range due to the thermal action on the free stream and is equal to the ratio of the initial power of the engine to the sum of the engine power under the action on the flow and the energy of the fuel spent on generation of the absorbed radiation Q. Another important characteristic is the ratio of the fuel spent on generation of the radiation energy g_{TQ} to the fuel spent on thrust generation g_{TR}:

$$z = \frac{g_{TQ}}{g_{TR}} = \frac{1-\varepsilon}{\eta_Q} \frac{\bar{F}_0 \bar{K}}{(k-1) c_x^0 \mathrm{M}_\infty^0 / 2} \frac{I a_\infty}{\mathrm{Hu}}.$$

Estimates. To obtain estimates, we used the following initial parameters typical of HFV with a hydrogen-powered air-breathing engine: $K_0 = 4$, $\alpha = 2$–$3°$, $c_y^0 = 0.1$, $\xi = 1$, and $\beta = \mathrm{Hu}/(I a_\infty) \approx 10$. The parameter η_Q was assumed to be equal to 0.2, since the flow-control method considered is little effective for lower values of this parameter. We obtain $\bar{c}_{x2}^0 = 0.8$. The lift-to-drag ratios for air heating equal to $\varepsilon = 0.1$, 0.2, 0.3, and 0.4 are

$1/\bar{K} = 0.28$, 0.36, 0.44, and 0.52, respectively. These results are close to the data of [2]. For $\varepsilon = 0.4$ and $\mathrm{M}^0_\infty = 10$ and 15, we have $\eta_L = 1.22$ and 1.39, respectively. For $\mathrm{M}^0_\infty = 10$, we have $z \approx 0.4$, which indicates that a significant energy is spent on flow control.

From the condition $\eta_L = 1$, we determine the minimum Mach number at which incoming flow heating is reasonable:

$$\mathrm{M}^0_{\infty\,\min} = \frac{\bar{F}_0}{(k-1)c^0_{x2}\eta_Q/2}\,\frac{Ia_\infty}{\mathrm{Hu}}.$$

For conditions of our estimates, we obtain $\mathrm{M}^0_{\infty\,\min} \simeq 6$.

Limiting Relations. As $\varepsilon \to 0$, we obtain

$$\bar{K} \to \frac{1}{\alpha K_0}, \qquad z \to \frac{\bar{F}_0\bar{K}}{\eta_Q\beta(k-1)c^0_x\mathrm{M}^0_\infty/2}, \qquad \eta_L \to \frac{\bar{K}}{1+z}.$$

As $\mathrm{M}^0_\infty \to \infty$, correspondingly, we have $z \to 0$ and $\eta_L \to \bar{K}$, i.e., as the flight velocity increases, the flow-control efficiency also increases, and the fraction of energy spent on flow control decreases.

Flight Range of a Missile-Shaped Body on the Active Trajectory. The equations of motion have the following form:

$$L = V_\infty t, \quad \dot{m} = -R/I - Q/(\eta_Q\mathrm{Hu}), \quad R = X.$$

The relative flight length $\bar{L} = L/L_0$ is determined by the relations

$$\bar{L} = \eta_L\Big(1 - \frac{\Delta\bar{m}_k}{g_T}\Big), \quad \frac{1}{\eta_L} = \frac{RV_\infty + Q/\eta_Q}{R_0V_\infty} = \varepsilon\xi + \frac{1-\varepsilon}{\eta_Q}\,\frac{1}{c^0_x(k-1)\mathrm{M}^0_\infty/2}\,\frac{Ia_\infty}{\mathrm{Hu}}.$$

Here g_T is the initial relative amount of fuel in the missile and $\Delta\bar{m}_k$ is the relative mass of the energy-source construction.

For the variant considered, the minimum Mach number at which flow heating is reasonable is determined by the expression

$$\mathrm{M}^0_{\infty\,\min} = \frac{1}{(k-1)c^0_x\eta_Q/2}\,\frac{Ia_\infty}{\mathrm{Hu}}.$$

Estimates. To obtain estimates, the following initial parameters are used: $c^0_x = 0.3$, $\eta_Q = 0.2$, $\beta = 30$, $\varepsilon = 0.4$, and $\xi = 1$. In calculations we obtained $\mathrm{M}^0_{\infty\,\min} = 2.8$; the coefficient of increasing length was $\eta_L = 1.22$ for $\mathrm{M}^0_\infty = 4$.

Flight with Acceleration. The equations of motion of the center of mass of an aerospace plane have the following form in the two-dimensional case:

$$\frac{V^0}{g}\,\dot{w} = \frac{R}{mg} - \frac{1}{K}\,\frac{Y}{mg} - \sin\theta = n_V, \quad \dot{m} = -\frac{R}{I} - \frac{Q}{\eta_Q\mathrm{Hu}}, \quad \frac{Y}{mg} = \cos\theta(1-w^2).$$

Here $V^0 = \sqrt{gr_e}$ is the orbital velocity, r_e is the Earth radius, $w = V/V^0$ is the relative velocity of the flying vehicle, θ is the slope of the FV trajectory, and n_V is the relative longitudinal acceleration. The relative consumption of fuel per unit increment of the relative velocity is calculated by the formula

$$\frac{1}{\bar{m}}\,\frac{dg_T}{dw} = \frac{V^0}{I}\Big[1 + \frac{1}{Kn_V}\,\cos\theta(1-w^2) + \sin\theta\Big] + \frac{2\bar{F}_0c_pT_\infty(1-\varepsilon)}{\eta_Q\mathrm{Hu}}\,\frac{q_\infty}{m_0g/S}\,\frac{1}{n_V\bar{m}w},$$

where $\bar{m} = m/m_0$ (m_0 is the initial FV mass and m is the FV mass at the time t) and $g_T = 1 - \bar{m}$.

Estimates. As an example, we estimate the relative consumption of fuel for the following parameters: $\mathrm{M}^0_\infty = 10$, $q^0_\infty = 0.5$ bar, $n_V = 0.5$, $K_0 = 3.5$, $\varepsilon = 0.4$, $\bar{K} = 2$, $I = 1.5\cdot 10^4$ m/sec, $\bar{F}_0 = 0.1$, $q^0_\infty/(m_0g/S) = 10$, and $\eta_Q = 0.2$. As a result, we obtain the estimate

$$\Big(\frac{1}{\bar{m}}\,\frac{dg_T}{dw}\Big)_Q \approx \frac{3}{4}\Big(\frac{1}{\bar{m}}\,\frac{dg_T}{dw}\Big)_0,$$

where the quantities with the subscripts Q and 0 refer to variants with and without flow control, respectively. It was found numerically that the amount of fuel spent on launching an aerospace plane to the Earth orbit with an altitude of 200 km under a thermal action on the incoming flow within the Mach number range $\mathrm{M}_\infty = 6$–17 may be reduced approximately by 3%.

The authors are grateful to Dr. P. K. Tret'yakov for fruitful discussions during the work.

REFERENCES

1. Yu. P. Gun'ko, I. I. Mazhul', and L. A. Chirkova, "Some estimates of the lift-to-drag ratio of hypersonic flying vehicles with air-breathing engines on the basis of experimental data," in: *Aircraft Engineering* (collected scientific papers) [in Russian], Nos. 5–6, (1995), pp. 34–38.
2. G. G. Chernyi, "The impact of electromagnetic energy addition to air near the flying body on its aerodynamics characteristics," in: *Proc. of the 2nd Weakly Ionized Gases Workshop* (Norfolk, VA, April 24–25, 1998), S. n., Nortfolk (1998), pp. 1–31.
3. L. Mirabo, Yu. P. Raizer, and M. N. Schneider, "Calculation and theory of similarity of an experiment simulating the "AIR-SPIKE" effect in hypersonic aerodynamics," *Teplofiz. Vys. Temp.*, **36**, No. 2, 304–309 (1998).
4. S. V. Guvernyuk and A. B. Samoilov, "Supersonic flow control with a pulsed thermal source," *Pis'ma Zh. Tekh. Fiz.*, **23**, No. 9, 1–8 (1997).
5. P. Yu. Georgievsky and V. A. Levin, "Unsteady effects for a supersonic flow past a pulsing energy source of high power," in: *Proc. of the Int. Conf. on the Methods of Aerophysical Research* (Novosibirsk, June 29–July 3, 1998), Part 2, Inst. Theor. Appl. Mech., Sib. Div., Russian Acad. of Sci., Novosibirsk (1998), pp. 58–64.
6. G. A. Luk'yanov, "Drag and heat exchange of a body in a supersonic flow with a planar source of energy upstream of the body," *Pis'ma Zh. Tekh. Fiz.*, **24**, No. 24, 76–82 (1998).
7. G. A. Luk'yanov, "Drag of a body in a supersonic flow with an isobaric energy-release region upstream of the body," *Pis'ma Zh. Tekh. Fiz.*, **25**, No. 1, 68–74 (1999).
8. V. Yu. Borzov, I. V. Rybka, and A. S. Yur'ev, "Effect of local energy addition to a hypersonic flow on the drag of bodies with different bluntness," *Inzh.-Fiz. Zh.*, **67**, Nos. 5–6, 355–361 (1994).
9. P. K. Tret'yakov, A. F. Garanin, G. P. Grachev, et al., "Supersonic flow control by a powerful optical pulsed discharge," *Dokl. Ross. Akad. Nauk*, **351**, No. 3, 339–340 (1996).
10. V. I. Artem'ev, V. I. Bergel'son, I. V. Nemchinov, et al., "Effect of a 'thermal spike' upstream of a blunted body in a supersonic flow," *Dokl. Ross. Akad. Nauk*, **310**, No. 1, 47–50 (1990).
11. V. V. Vitkovskii, L. P. Grachev, N. N. Gritsov, et al., "Unsteady flow around bodies by a supersonic air flow heated by a longitudinal electric discharge," *Teplofiz. Vys. Temp.*, **28**, No. 6, 1156–1163 (1990).
12. V. A. Belokon', O. V. Rudenko, and R. V. Khokhlov, "Aerodynamic phenomena in a supersonic flow around a laser beam," *Akust. Zh.*, **23**, No. 4, 632–634 (1977).
13. P. K. Tret'yakov and V. I. Yakovlev, "Formation of a quasi-steady supersonic flow with a pulsed periodic plasma source," *Pis'ma Zh. Tekh. Fiz.*, **22**, No. 24, 30–33 (1998).
14. P. K. Tret'yakov and V. I. Yakovlev, "Wave structure in a supersonic flow with laser energy addition," *Dokl. Ross. Akad. Nauk*, **365**, No. 1, 58–60 (1999).
15. P. K. Tretyakov, V. I. Yakovlev, and V. N. Zudov, "Gasdynamic structure of a supersonic flow around a heat source," in: *Rec. of* 19*th Int. Congress on Instrument. in Aerospace Simulation Facilities* (Cleveland, Ohio, August 27–30, 2001), IEEE, Cleveland (2001), pp. 70–81.
16. V. N. Zudov, "Evolution of a wake behind a pulsed periodic source of energy," *Pis'ma Zh. Tekh. Fiz.*, **27**, No. 12, 81–87 (2001).
17. L. V. Gogish and S. G. Dashevskaya, "Supersonic Nonuniform flow around a tapered airfoil," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 180–183 (1990).
18. F. Bartlma, *Gasdynamik der Verbrennung*, Springer-Verlag, Wien (1975).
19. V. Yu. Borzov, I. V. Rybka, and A. S. Yur'ev, "Evaluation of energy consumption with decreasing body drag in a supersonic gas flow," *Inzh.-Fiz. Zh.*, **63**, No. 6, 659–664 (1992).
20. V. M. Mikhailov, G. N. Moskalets, I. V. Rybka, et al., "Method of increasing efficiency of action on a supersonic flow with implementation of the energy method of flow control," in: *Basic Research for Hypersonic Technologies*, Proc. All-Union Conf. (Zhukovskii, October 20–23, 1998), Aerohydrodynamic Center, Zhukovskii (1998), pp. 1–12.